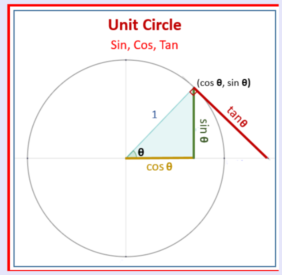
Trigonometry

Lecture 45

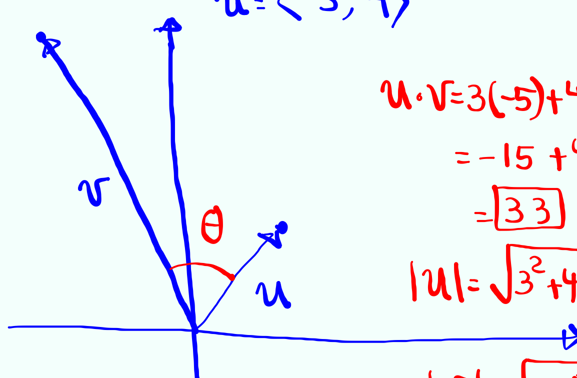


Feb 19-8:47 AM

Find the angle between $u = 3i + 4j$ and $v = -5i + 12j$.

$u = \langle 3, 4 \rangle$

$v = \langle -5, 12 \rangle$



$u \cdot v = 3(-5) + 4(12)$

$= -15 + 48$

$= \boxed{33}$

$|u| = \sqrt{3^2 + 4^2} = 5$

$|v| = \sqrt{(-5)^2 + 12^2} = 13$

$$\cos\theta = \frac{u \cdot v}{|u||v|} = \frac{33}{5 \cdot 13}$$

$$\cos\theta = \frac{33}{65}$$

$$\theta = \cos^{-1}\left(\frac{33}{65}\right) \approx \boxed{59^\circ}$$

Nov 20-10:34 AM

Find a unit vector in the direction of

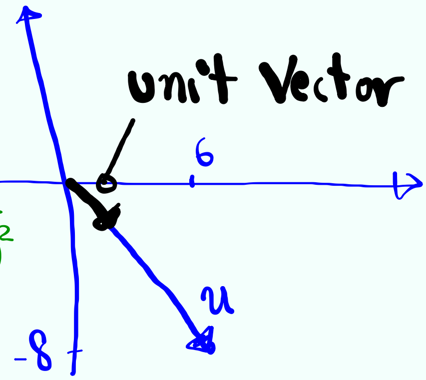
$u = 6i - 8j$

$u = \langle 6, -8 \rangle$

$v = \frac{1}{|u|} u$

$|u| = \sqrt{6^2 + (-8)^2}$

$= 10$



$v = \frac{1}{10} u = \frac{1}{10}\langle 6, -8 \rangle = \boxed{\langle .6, -.8 \rangle}$

Nov 20-10:38 AM

Given $u = \langle 2, 6 \rangle$ $v = \langle 5, 1 \rangle$

Find $\text{Proj}_v u$

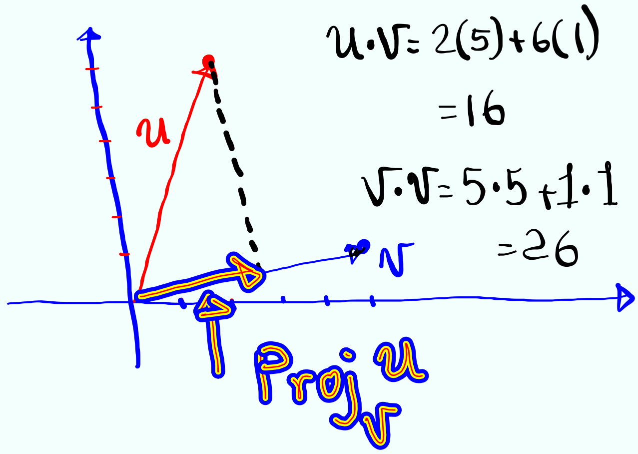
Orthogonal Proj.

$u \cdot v = 2(5) + 6(1)$

$= 16$

$v \cdot v = 5 \cdot 5 + 1 \cdot 1$

$= 26$



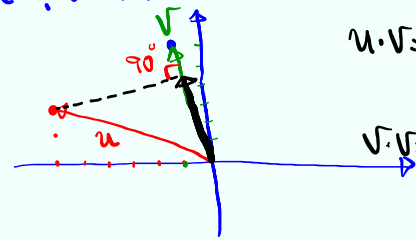
$\text{Proj}_v u = \frac{u \cdot v}{v \cdot v} v$

$= \frac{16}{26}\langle 5, 1 \rangle = \frac{8}{13}\langle 5, 1 \rangle = \langle \frac{40}{13}, \frac{8}{13} \rangle$

Nov 20-10:42 AM

$u = \langle -6, 2\rangle \qquad v = \langle -1, 6\rangle$

1) Draw u & v in standard position.



$u \cdot v = (-6)(-1) + 2(6)$
$= 18$

$v \cdot v = (-1)(-1) + 6(6)$
$= 37$

2) Display $\text{Proj}_v u$ on same graph

3) Find $\text{Proj}_v u = \dfrac{u \cdot v}{v \cdot v} v = \dfrac{18}{37}\langle -1, 6\rangle$

$= \left\langle \dfrac{-18}{37}, \dfrac{108}{37} \right\rangle$

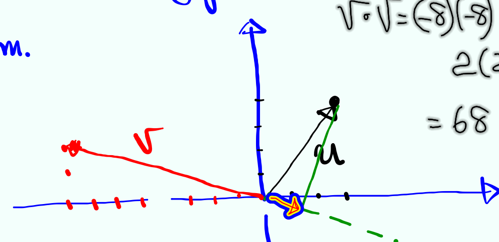
Nov 20-10:47 AM

$u \cdot v = 3(-8) + 4 \cdot 2$
$= -24 + 8$
$= -16$

$u = \langle 3, 4\rangle \quad , \quad v = \langle -8, 2\rangle$

1) Draw u, v, and $\text{Proj}_v u$ in the Same Coordinate System.

$v \cdot v = (-8)(-8) + 2(2)$
$= 68$



$4 \quad \cancel{8}$

2) Find $\text{Proj}_v u = \dfrac{u \cdot v}{v \cdot v} v = \dfrac{\cancel{-16}}{\cancel{68}}\langle -8, 2\rangle$

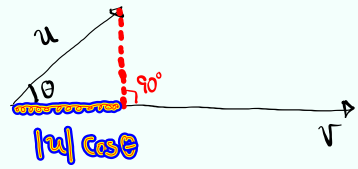
$\dfrac{\cancel{34}}{17}$

$=$

$= \left\langle \dfrac{32}{17}, \dfrac{-8}{17} \right\rangle$

Nov 20-10:54 AM

Idea behind $\text{Proj}_{v}\, u$



1) Find a unit vector in direction of v

$$\frac{1}{|v|}v$$

2) Drop a perpendicular line from terminal point of u on v.

$$\cos\theta = \frac{u\cdot v}{|u|\,|v|}$$

3) Multiply that unit vector in the direction of v by $|u|\cos\theta$

$$|u|\cos\theta\frac{1}{|v|}v = \cancel{|u|}\cdot\frac{u\cdot v}{\cancel{|u|}|v|}\cdot\frac{1}{|v|}v$$

$$= \frac{u\cdot v}{|v|^2}v = \boxed{\frac{u\cdot v}{v\cdot v}v}$$

$\text{Proj}_{v}\, u$

Nov 20-11:04 AM

Given $u = \langle 2,-5\rangle$, $v = \langle 5,2\rangle$

1) $u+v = \langle 7,-3\rangle$

2) $u-v = \langle -3,-7\rangle$

3) $|u| = \sqrt{2^2+(-5)^2} = \sqrt{29}$

4) $u\cdot v$

$= 2\cdot 5 + (-5)\cdot 2$

$= 10 - 10 = \boxed{0}$

5) $2u - 3v = 2\langle 2,-5\rangle - 3\langle 5,2\rangle$

$= \langle 4,-10\rangle - \langle 15,6\rangle = \langle -11,-16\rangle$

6) Find the angle between u & v.

$$\cos\theta = \frac{u\cdot v}{|u|\,|v|}\qquad \cos\theta = \frac{0}{\sqrt{29}\sqrt{29}} = \frac{0}{29} = 0$$

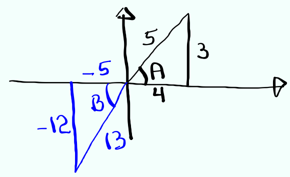
$\cos\theta = 0$

$\theta = 90^\circ$

Nov 20-11:11 AM

$\sin A = \frac{3}{5}$ A is in QI

$\cos B = \frac{-5}{13}$ B is in QIII



$\sin(A + B)$

$= \sin A \cos B + \cos A \sin B$

$= \frac{3}{5} \cdot \frac{-5}{13} + \frac{4}{5} \cdot \frac{-12}{13}$

$= \frac{-15-48}{65} = \boxed{\frac{-63}{65}}$

$\cos(A - B)$

$= \cos A \cos B + \sin A \sin B$

$= \frac{4}{5} \cdot \frac{-5}{13} + \frac{3}{5} \cdot \frac{-12}{13}$

$= \frac{-20-36}{65} = \boxed{\frac{-56}{65}}$

$\tan 2A = \frac{2\tan A}{1-\tan^2 A} = \frac{2 \cdot \frac{3}{4}}{1-\left(\frac{3}{4}\right)^2} = \frac{\frac{3}{2}}{1-\frac{9}{16}} = \frac{24}{16-9}$

LCD = 16 $= \frac{24}{7}$

Nov 20-11:18 AM

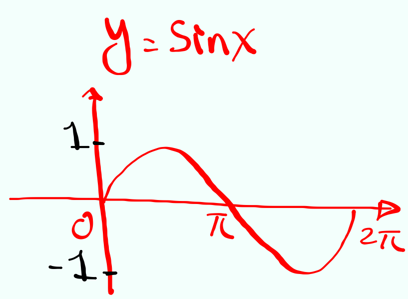
Graph one period of

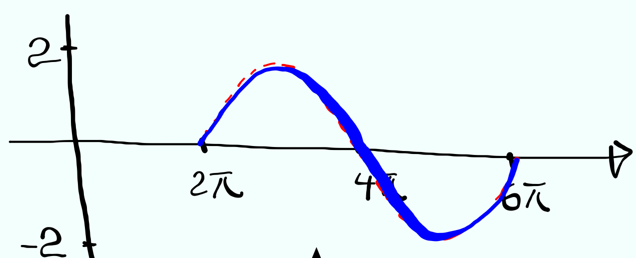
$y = 2\sin\left(\frac{1}{2}x - \pi\right)$

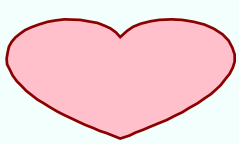
$0 \le \frac{1}{2}x - \pi \le 2\pi$

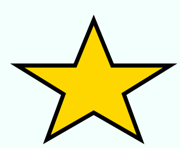
$\pi \le \frac{1}{2}x \le 3\pi$

$2\pi \le x \le 6\pi$





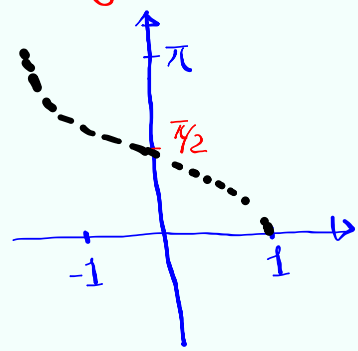




Nov 20-11:27 AM

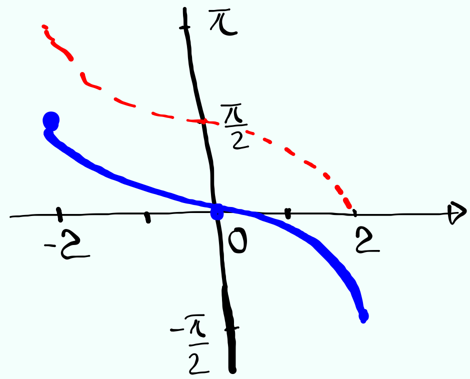
Graph $y = -\frac{\pi}{2} + \cos^{-1}\left(\frac{1}{2}x\right)$

$y = \cos^{-1} x$



$-1 \le \frac{1}{2}x \le 1$

$-2 \le x \le 2$



Nov 20-11:31 AM

Solve $\sin 2x \cos x - \cos x = 0$

General solutions in radians

$\cos x(\sin 2x - 1) = 0$

$\cos x = 0$

$\boxed{\frac{\pi}{2} + k \cdot \pi}$

$\sin 2x - 1 = 0$

$\sin 2x = 1$

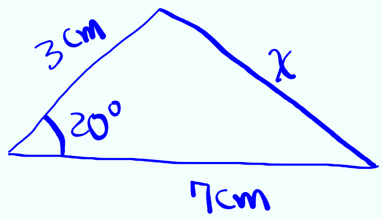
R.A. $\frac{\pi}{2}$

$2x = \frac{\pi}{2} + k \cdot 2\pi$

$x = \boxed{\frac{\pi}{4} + k \cdot \pi}$

Nov 20-11:35 AM

find x



Law of Cosines

$x^2 = 3^2 + 7^2 - 2 \cdot 3 \cdot 7 \cos 20°$

$x^2 = 18.5$

$x \approx 4.3\text{cm}$

we round to whole #
because other sides
are whole #

$x \approx 4\text{cm}$

Nov 20-11:40 AM